RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

SECOND YEAR

B.A./B.SC. FOURTH SEMESTER (January – June) 2013

Mid-Semester Examination, March 2013

Date : 04/03/2013

MATHEMATIVCS (Honours)

Time : 2 pm – 4 pm

Paper : IV

Full Marks : 50

[Use separate Answer Books for each group]

Group – A

1. Select the correct alternative in each of the following : [6×2]

   i) Suppose $\mathcal{P}$(N) denotes the power set of N. Consider the matric 'd' on $\mathcal{P}$(N) defined by

   $$d(A,B)=\begin{cases} 0, \text{ if } A\Delta B=\phi \\ \frac{1}{m}, \text{where m is the smallest element of } A\Delta B \end{cases}$$

   Let A = {{1}, {1,2}, {1,2,3}, {1,2,3,4}}. Then—

   a) diam (A) = 1 b) diam (A) = $\frac{1}{2}$ c) diam (A) = $\frac{1}{3}$ d) diam (A) = $\frac{1}{4}$

   ii) Let (X,d) be a metric space; $a,b\in X$ and d(a,b) = 2r. Then—

   a) Any two open balls, one with centre a and the other with centre b always intersect

   b) $\{x\in X: d(a,x)\le r\}\cap\{x\in X: d(b,x)\}\le r\neq\phi$

   c) Any open set containing a must contain b.

   d) None of the above

   iii) In a metric space,

   a) Only closed sets are G_δ b) All countable sets are G_δ

   c) All open sets are G_δ d) If a subset is G_S then it can't be F_σ

   iv) In R with usual matric,

   a) Q–N is dense

   b) Any uncountable subset is dense

   c) There does not exist two countable, disjoint, dense sets

   d) A countable subset of R–Q can't be dense

   v) In R with usual matric,

   a) Q is the boundary of some subset of R

   b) R–Q is the boundary of some subset of R

   c) {1,2} is the boundary of at least three different subsets of R

   d) If $A,B\subseteq R$ then $Bd(A\cup B)=BdA\cup BdB$

   vi) In R with usual metric,

   a) Each point of $A=\left\{\frac{1}{n}: n=1,2,...\right\}$ is an isolated point of A

   b) Only finitely many subsets of R have infinitely many isolated points

   c) If $A\subseteq R$ and A has an isolated point then R–A has no isolated point

   d) The only isolated point of Q is '0'

2. Answer **any two** questions : [2×2½]

   a) Let G be an open subgroup of (R,+). Show that G = R.

   b) In a metric space, prove that every closed ball is a closed set.

c) Let (X,d) be a metric space and $A \subseteq X$. If $\bar{A}$ is the smallest closed set containing A then prove that $\bar{A} = X$ iff every non-empty open set intersects A.

3. Answer **<u>any two</u>** questions : [2×4]

a) Let $D \subset \mathbb{R}$ and for each $n \in \mathbb{N}, f_n : D \to \mathbb{R}$ is continuous on D. If the sequence $\{f_n\}_{n\in\mathbb{N}}$ converges uniformly on D to a function f, then prove that f is continuous on D

b) For each $n \in \mathbb{N}$, let $f_n : [0,\infty) \to \mathbb{R}$ be defined by $f_n(x) = \dfrac{nx}{1+nx}, x \in [0,\infty)$. Show that $\{f_n\}_{n\in\mathbb{N}}$ is not uniformly convergent on $[0,\infty)$ but is uniformly convergent on $[a,\infty)$ if a > 0.

c) Let $\{f_n\}_{n\in\mathbb{N}}$ and $\{g_n\}_{n\in\mathbb{N}}$ be sequences of bounded functions which converge uniformly on E to the functions f and g respectively. Prove that $\{f_n g_n\}_{n\in\mathbb{N}}$ converges uniformly on E to the function fg.

<u>Group – B</u>

4. Answer **<u>any three</u>** : [3×5]

a) Solve $(1-x^2)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} - y = x(1-x^2)$ given that y = x is a solution of its reduced equation.

b) Find the eigen-values and eigen-functions of $\dfrac{d}{dx}\left(x\dfrac{dy}{dx}\right) + \dfrac{\lambda}{x}y = 0\ (\lambda > 0)$ with boundary conditions $y(1) = 0$ and $y(e^{\pi}) = 0$.

c) Solve the system $\left(D \equiv \dfrac{d}{dt}\right)$

$$\left.\begin{aligned}(D^2+4)x + y &= te^{3t}\\ (D^2+1)y - 2x &= \cos^2 t\end{aligned}\right\}$$

d) Solve : $\dfrac{dx}{x^2+a^2} = \dfrac{dy}{xy-az} = \dfrac{dz}{xz+ay}$

e) Solve : $(2x^2 + 2xy + 2xz^2 + 1)dx + dy + 2zdz = 0$

5. Answer **<u>any one</u>** : [5]

a) Find the whole area included between the curve $y^2(a-x) = x^2(a+x)$ and its asymptote, a > 0.

b) The arc of the cardioide $r = a(1+\cos\theta)$ specified by $-\dfrac{\pi}{2} \le \theta \le \dfrac{\pi}{2}$, is rotated about the line $\theta = 0$. Find the area of the generated surface of revolution.

6. Answer **<u>any one</u>** : [5]

a) Prove that if the curves $ax^2 + by^2 = 1$ and $Ax^2 + By^2 = 1$ intersect at right angles, then $\dfrac{1}{A} - \dfrac{1}{a} = \dfrac{1}{B} - \dfrac{1}{b}$.

b) Tangents are drawn from the origin to the curve y = sinx. Show that their points of contact lie on the curve $x^2y^2 = x^2 - y^2$.